

STUDY EDITION

MATHEMATICS: ANALYSIS AND APPROACHES

Higher Level formula booklet, annotated page for page.

Practise with this copy and the official booklet will already
feel familiar — same topics, same order, same page.



Level

Higher Level (HL)

First examination

2021

Topics

1 Number and algebra · 2 Functions · 3 Geometry and trigonometry ·
4 Statistics and probability · 5 Calculus

How to use it

Every topic band, syllabus reference and page break matches the
original, so the page you learn here is the page you turn to there.

Printing

A4, scale 100%, background graphics on. One sheet prints as one
page.

Annotated study edition — always sit the exam with the official booklet your school issues.

This page is intentionally blank.

INDEX

CONTENTS

T1	Number and algebra — HL	7 BLOCKS	2
T2	Functions — HL	4 BLOCKS	3
T3	Geometry and trigonometry — HL	9 BLOCKS	4
T4	Statistics and probability — HL	9 BLOCKS	7
T5	Calculus — HL	9 BLOCKS	9

Reading the tables. The left column is the syllabus reference — the same number your teacher and the exam paper use. The middle column names the result; the right column is the formula itself, with its conditions underneath.

Prior learning. Topic 3 opens with the prior-learning results (areas, volumes, distance and midpoint) that the course assumes rather than re-teaches. They sit on page 4 because that is where the original puts them.

TOPIC 1 NUMBER AND ALGEBRA — HL

1.2	The n th term of an arithmetic sequence	$u_n = u_1 + (n - 1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n - 1)d)$ $S_n = \frac{n}{2}(u_1 + u_n)$
1.3	The n th term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$
1.8	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1 - r}, \quad r < 1$
1.4	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest.
1.5	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$ where $a > 0, b > 0, a \neq 1$
1.7	Exponents and logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
	Exponential and logarithmic functions	$a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$
1.9	Binomial theorem $n \in \mathbb{N}$	$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$ ${}^nC_r = \frac{n!}{r!(n-r)!}$

1.10	Combinations	${}^nC_r = \frac{n!}{r!(n-r)!}$
	Permutations	${}^nP_r = \frac{n!}{(n-r)!}$
	Extension of binomial theorem, $n \in \mathbb{Q}$	$(a+b)^n = a^n \left(1 + n\left(\frac{b}{a}\right) + \frac{n(n-1)}{2!}\left(\frac{b}{a}\right)^2 + \dots \right)$
1.12	Complex numbers	$z = a + bi$
1.13	Modulus–argument (polar) and exponential (Euler) form	$z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$
1.14	De Moivre's theorem	$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta) = r^n e^{in\theta} = r^n \operatorname{cis} n\theta$

TOPIC 2 FUNCTIONS — HL

2.1	Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2.6	Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
2.7	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$
2.12	Sum and product of the roots of polynomial equations of the form $\sum_{r=0}^n a_r x^r = 0$	Sum is $-\frac{a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

TOPIC 3 GEOMETRY AND TRIGONOMETRY — HL

PRIOR LEARNING PRIOR LEARNING — HL

	Area of a parallelogram	$A = bh$ where b is the base, h is the height
	Area of a triangle	$A = \frac{1}{2}(bh)$ where b is the base, h is the height
	Area of a trapezoid	$A = \frac{1}{2}(a + b)h$ where a and b are the parallel sides, h is the height
	Area of a circle	$A = \pi r^2$ where r is the radius
	Circumference of a circle	$C = 2\pi r$ where r is the radius
	Volume of a cuboid	$V = lwh$ where l is the length, w is the width, h is the height
	Volume of a cylinder	$V = \pi r^2 h$ where r is the radius, h is the height
	Volume of a prism	$V = Ah$ where A is the area of cross-section, h is the height
	Area of the curved surface of a cylinder	$A = 2\pi rh$ where r is the radius, h is the height
	Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
	Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
3.1	Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
	Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
	Volume of a right-pyramid	$V = \frac{1}{3}Ah$ where A is the area of the base, h is the height

	Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$ where r is the radius, h is the height
	Area of the curved surface of a cone	$A = \pi r l$ where r is the radius, l is the slant height
	Volume of a sphere	$V = \frac{4}{3}\pi r^3$ where r is the radius
	Surface area of a sphere	$A = 4\pi r^2$ where r is the radius
3.2	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2}ab \sin C$
3.4	Length of an arc	$l = r\theta$ where r is the radius, θ is the angle measured in radians
	Area of a sector	$A = \frac{1}{2}r^2 \theta$ where r is the radius, θ is the angle measured in radians
3.5	Identity for $\tan \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
3.6	Pythagorean identity	$\cos^2 \theta + \sin^2 \theta = 1$
	Double angle identities	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta$
3.9	Reciprocal trigonometric identities	$\sec \theta = \frac{1}{\cos \theta}$ $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
	Pythagorean identities	$1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

3.10	Compound angle identities	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
	Double angle identity for $\tan$	$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
3.12	Magnitude of a vector	$ v = \sqrt{v_1^2 + v_2^2 + v_3^2}, \text{ where } v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
3.13	Scalar product	$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3, \text{ where } v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $v \cdot w = v w \cos \theta, \text{ where } \theta \text{ is the angle between } v \text{ and } w$
	Angle between two vectors	$\cos \theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{ v w }$
3.14	Vector equation of a line	$r = a + \lambda b$
	Parametric form of the equation of a line	$x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$
	Cartesian equations of a line	$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
3.16	Vector product	$v \times w = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}, \text{ where } v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $ v \times w = v w \sin \theta, \text{ where } \theta \text{ is the angle between } v \text{ and } w$
	Area of a parallelogram	$A = v \times w \text{ where } v \text{ and } w \text{ form two adjacent sides of a parallelogram}$
3.17	Vector equation of a plane	$r = a + \lambda b + \mu c$
	Equation of a plane (using the normal vector)	$r \cdot n = a \cdot n$
	Cartesian equation of a plane	$ax + by + cz = d$

TOPIC 4 STATISTICS AND PROBABILITY — HL

4.2	Interquartile range	$IQR = Q_3 - Q_1$
4.3	Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$, where $n = \sum_{i=1}^k f_i$
4.5	Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
	Complementary events	$P(A) + P(A') = 1$
4.6	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A) P(B)$
4.7	Expected value of a discrete random variable X	$E(X) = \sum_{i=1}^k x_i P(X = x_i)$
4.8	Binomial distribution $X \sim B(n, p)$	Mean and variance of the distribution.
	Mean	$E(X) = np$
	Variance	$\text{Var}(X) = np(1 - p)$
4.12	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$
4.13	Bayes' theorem	$P(B A) = \frac{P(B) P(A B)}{P(B) P(A B) + P(B') P(A B')}$ $P(B_i A) = \frac{P(B_i) P(A B_i)}{P(B_1)P(A B_1) + P(B_2)P(A B_2) + P(B_3)P(A B_3)}$

4.14	Variance σ^2	$\sigma^2 = \frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^k f_i x_i^2}{n} - \mu^2$
	Standard deviation σ	$\sigma = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}}$
	Linear transformation of a single random variable	$E(aX + b) = a E(X) + b$ $\text{Var}(aX + b) = a^2 \text{Var}(X)$
	Expected value of a continuous random variable X	$E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$
	Variance	$\text{Var}(X) = E[(X - \mu)^2] = E(X^2) - [E(X)]^2$
	Variance of a discrete random variable X	$\text{Var}(X) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2$
	Variance of a continuous random variable X	$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

TOPIC 5 CALCULUS — HL

5.12	Derivative of $f(x)$ from first principles	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$
5.3	Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
5.6	Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
	Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
	Chain rule	$y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
	Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
	Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
5.15	Standard derivatives	
	$\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$
	$\sec x$	$f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$
	$\operatorname{cosec} x$	$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$
	$\cot x$	$f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$
	a^x	$f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$
	$\log_a x$	$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$
	$\arcsin x$	$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
	$\arccos x$	$f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$
	$\arctan x$	$f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$

5.9	Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
	Distance travelled from t_1 to t_2	$\text{distance} = \int_{t_1}^{t_2} v(t) dt$
	Displacement from t_1 to t_2	$\text{displacement} = \int_{t_1}^{t_2} v(t) dt$
5.5	Integral of x^n	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
	Area between a curve $y = f(x)$ and the x -axis, where $f(x) > 0$	$A = \int_a^b y dx$
5.10	Standard integrals	$\int \frac{1}{x} dx = \ln x + C$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$
5.15	Standard integrals, continued	$\int a^x dx = \frac{1}{\ln a} a^x + C$ $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C, \quad x < a$
5.16	Integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$ or $\int u dv = uv - \int v du$

5.11	Area of region enclosed by a curve and x -axis	$A = \int_a^b y \, dx$
5.17	Area of region enclosed by a curve and y -axis	$A = \int_c^d x \, dy$
	Volume of revolution about the x or y -axes	$V = \int_a^b \pi y^2 \, dx$ or $V = \int_c^d \pi x^2 \, dy$
5.18	Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n) ; \quad x_{n+1} = x_n + h$ where h is a constant (step length)
	Integrating factor for $y' + P(x)y = Q(x)$	$e^{\int P(x) \, dx}$
5.19	Maclaurin series	$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots$
	Maclaurin series for special functions	$e^x = 1 + x + \frac{x^2}{2!} + \dots$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$